TEMPERATURE AND THERMAL FLUXES AT AN ELASTOHYDRODYNAMIC CONTACT

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UDC 532.516

Equations are derived for the temperature, thermal fluxes, and frictional stresses at elastohydrodynamic contacts.

Temperature and friction largely determine the efficiency of elastohydrodynamic (EHD) contacts, together with their energy losses. Viscous dissipation in a thin film of lubricant may lead to the development of high temperatures, a reduction in film thickness, and disruption of the normal operation of the contact. Jamming at the contact may also arise from thermal causes [1]. Measurements of the temperature of an EHD contact [2] based on infrared radiation showed that the surface temperatures might vary considerably, while the maximum temperature of the film might be greater still and rise as high as 360°C. The experiments of [2] were complicated and failed to determine the temperature, thermal fluxes, and coefficient of friction simultaneously. A theoretical solution may be derived from the equations of a lubricating film.

Let us consider the flow of a viscous liquid between two rotating elastic cylinders (Fig. 1). Let u_1 and u_2 be the linear velocities of the surfaces of the cylinders and T_1 and T_2, their temperatures. The cylinders are pressed one against the other by a linear load q, which gives a maximum Hertz pressure p_0 and a contact width b. The film thickness h may be regarded as constant for $p_0 \geqslant 5 \cdot 10^8$ N/m² [3] and calculated in the manner of [4]. The Reynolds number $Re \ll 1$ and $b \gg h$, so that the equation of motion of the lubricant takes the form

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}, \quad \frac{\partial p}{\partial y} = 0. \tag{1}$$

The lubricant is regarded as a Newtonian liquid with viscosity μ dependent upon the pressure p and temperature T:

$$\mu = \mu_0 \exp\left[\frac{\alpha p}{1+\beta p} - \delta (T - T_0)\right]. \tag{2}$$

An estimation of τ and the change Δτ in this quantity across the lubricant film gives

$$\frac{\Delta\tau}{\tau} = \frac{p_0 h^2}{\mu_1 \exp\left(\dfrac{\alpha p_0}{1+\beta p_0}\right)(u_2 - u_1)\, b}. \tag{3}$$

For the characteristic parameters of the EHD contact $p_0 = 10^9$ N/m², $\alpha = 1.2 \cdot 10^{-8}$ m²/N, $\beta = 0$, $u_2 - u_1 = 0.2$ m/sec, $b = 2 \cdot 10^{-4}$ m, $h = 10^{-7}$ m, $\mu_1 = 2 \cdot 10^{-3}$ (N•sec)/m², T = 100°C we obtain $\Delta\tau/\tau \sim 10^{-3}$, which allows us to regard τ as independent of y in the current problem. Let us consider the equation of energy

$$\rho c_v u \frac{\partial T}{\partial x} = -p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \tau \frac{\partial u}{\partial y} + k \frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where c_V is the specific heat and u, v are the velocity components. For $c_V = 1.6 \cdot 10^3$ J/(kg• deg), $\rho = 10^3$ kg/m³, u = 5 m/sec compressibility 0.1, k = 0.1 W/(m•deg), and temperature change ΔT = 100°C, the convective term and the heat evolution due to compression are of the order of 10^{12} W/m³, while the viscous dissipation and thermal conductivity are of the order of 10^{14} W/m³. Equation (4) thus transforms into an ordinary differential equation

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 4, pp. 687-690, April, 1977. Original article submitted April 12, 1976.

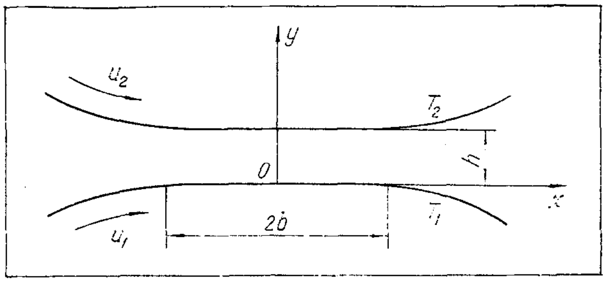

Fig. 1. Elastohydrodynamic contact between cylinders.

$$k\frac{d^2T}{dy^2}=-\frac{\tau^2}{\mu},\tag{5}$$

and the system for T and u assumes the following form in the dimensionless variables $t = \delta\cdot(T - T_0)$, $\eta = y/h$, $\xi = u/(u_2 - u_1)$:

$$\frac{d\xi}{d\eta}=A\exp t,\ \xi(0)=\frac{u_1}{u_2-u_1},\ \xi(1)=\frac{u_2}{u_2-u_1},\tag{6}$$

$$\frac{d^2t}{d\eta^2}=B\exp t,\ t(0)=t_1,\ t(1)=t_2.\tag{7}$$

Here

$$A=\frac{\tau h}{\mu_0\exp\left[\frac{\alpha p}{1+\beta p}\right](u_2-u_1)},\quad B=\frac{\delta\tau^2h^2}{\mu_0 k\exp\left[\frac{\alpha p}{1+\beta p}\right]}.$$

Reducing the order of (7) and integrating, we obtain the system of equations

$$\frac{2}{\sqrt{C_1}}\left(\mp\operatorname{arth}\frac{\sqrt{C_1-2B\exp t_2}}{\sqrt{C_1}}+\operatorname{arth}\frac{\sqrt{C_1-2B\exp t_1}}{\sqrt{C_1}}\right)=1,\tag{8}$$

$$\frac{A}{B}\left(\sqrt{C_1-2B\exp t_1}\mp\sqrt{C_1-2B\exp t_2}\right)=1,\tag{9}$$

where C_1 and τ are the unknowns, the minus sign is taken for a monotonic variation of t across the film, and the plus sign is taken if a maximum value of t appears. We may find an exact solution of (8) and (9):

$$C_1=2B\exp t_1+[A(\exp t_2-\exp t_1)+B/2A]^2,\tag{10}$$

$$\tau=\mu\frac{u_2-u_1}{h}\cdot\frac{\operatorname{arsh}\Lambda}{\Lambda\sqrt{1+\Lambda^2}},\tag{11}$$

where

$$\mu=\mu_0\exp\left[\frac{\alpha p}{1+\beta p}-\delta\left(\frac{T_1+T_2}{2}-T_0\right)\right],$$

$$\Lambda=\sqrt{\Lambda_0^2+\operatorname{sh}^2\left[\left(\frac{T_2-T_1}{4}\right)\delta\right]},$$

$$\Lambda_0=\sqrt{\frac{\mu\delta}{8k}}\,|u_2-u_1|.$$

Equation (11) gives the frictional forces at the EHD rolling and sliding contact for various surface temperatures. Under the condition

$$\frac{2}{\sqrt{C_1}}\operatorname{arth}\frac{\sqrt{C_1-2B\exp t_1}}{\sqrt{C_1}}<1$$

the temperature has a maximum inside the film:

$$T_{\max}=\frac{T_1+T_2}{2}+\frac{1}{\delta}\ln\left[\operatorname{ch}\frac{\delta(T_2-T_1)}{2}+\Lambda_0^2+\frac{1}{4\Lambda_0^2}\operatorname{sh}^2\left[\frac{\delta(T_2-T_1)}{2}\right]\right].\tag{12}$$

The thermal fluxes q_1 and q_2 are determined from (7):

$$q_1 = \frac{\tau (u_2 - u_1)}{2} \left[1 + \frac{1}{2\Lambda_0^2} \operatorname{sh} \frac{\delta (T_2 - T_1)}{2} \right], \tag{13}$$

$$q_2 = \frac{\tau (u_2 - u_1)}{2} \left[1 - \frac{1}{2\Lambda_0^2} \operatorname{sh} \frac{\delta (T_2 - T_1)}{2} \right]. \tag{14}$$

In order to be specific we consider that $T_2 > T_1$. We see that the flow of heat to the more heated body is smaller and may even change sign with increasing temperature difference. The solution is also applicable for an arbitrary dependence of $\mu(p)$, $T_1(x)$, and $T_2(x)$. For $T_1 = T_2 = T_0$ Eq. (11) was obtained in [5].

The velocity and temperature profiles are also determined from (6) and (7):

$$T = T_0 + \frac{1}{\delta} \ln \left\{ \frac{C_1}{2B \operatorname{ch}^2 [(y + C_2 h) \sqrt{C_1}/2h]} \right\}, \tag{15}$$

$$u = \frac{u_1 + u_2}{2} + \left\{ \frac{A^2}{B} (\exp t_2 - \exp t_1) + \frac{A}{B} \sqrt{C_1} \operatorname{th} \left[\frac{y + C_2 h}{h} \sqrt{C_1} \right] \right\} (u_2 - u_1), \tag{16}$$

where

$$C_2 = \pm \frac{2}{\sqrt{C_1}} \operatorname{arth} \frac{\sqrt{C_1 - 2B \exp t_1}}{\sqrt{C_1}}.$$

In the two-dimensional case, by directing the Oz axis along the normal to the Ox and Oy and allowing slip $v_2 - v_1$ along z, we may also obtain simple equations for the components of frictional force τ_x and τ_z. The result obtained for τ_x coincides with (11), while

$$\tau_z = \mu \frac{v_2 - v_1}{h} \cdot \frac{\operatorname{arsh} \Lambda}{\Lambda \sqrt{1 + \Lambda^2}},$$

except that in Λ_0 the quantity $|u_2 - u_1|$ is replaced by the modulus of the slip vector $|\bar{u}_2 - \bar{u}_1|$ where $\bar{u}_j = (u_j, v_j)$, $j = 1, 2$. This result was given in [6] for $T_1 = T_2 = T_0$. In Eqs. (13) and (14) instead of $\tau(u_2 - u_1)$ we have $\tau_x(u_2 - u_1) + \tau_z(v_2 - v_1)$. Equations (15) and (16) change accordingly. An experimental investigation into the EHD contact of a steel sphere with a sapphire plate was carried out in [2]. The maximum temperature averaged over the film thickness was 360°C. The maximum temperature on the surface of the sphere was 115°C. The pressure $p_0 = 1.034 \cdot 10^9$ N/m², $u_2 - u_1 = 1.39$ m/sec, $v_2 - v_1 = 0$, $\delta = 3.2 \cdot 10^{-2}$ deg^{-1}, $\mu_0 = 3.26 \cdot 10^{-2}$ N·sec/m², $\beta = 0$. The piezocoefficient $\alpha = 1.5 \cdot 10^{-8}$ m²/N at T = 115°C. The temperature on the surface of the sapphire plate was taken as equal to the average temperature of the liquid, i.e., 115°C. Using Eq. (12) we obtain $T_{max} \approx 340$°C.

NOTATION

x, y, coordinates; p, pressure; τ, tangential (shear) stress; μ, viscosity; α, β, piezocoefficients; μ_0, viscosity under normal conditions; T_0, a certain fixed temperature; $\Delta\tau$, change in τ across the film; μ_1 viscosity at the temperature of the surface and room pressure; c_v, specific heat; u, v, velocity components; ρ, density; k, thermal conductivity; ΔT, change in temperature; t, dimensionless temperature; η, dimensionless coordinate; ξ, dimensionless velocity; A, B, parameters of the problem; C_1, constant; Λ, Λ_0, dimensionless film-heating parameters; q_1, q_2, thermal fluxes; τ_x, τ_z, shear stress components.

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